Seminar exercise 4

1. Ohm's law for loss on lines (direct current) is

$$P_o = P_i - P_L$$
$$P_L = I^2 R$$
$$P_o = V_o I$$
$$R = \frac{2L\rho}{A}$$

where the definitions of the variables are:

P_o = the consumption of power in kW
P_i = the generation of power in kW
P_L = the loss in kW
I = current in amps
R = resistance of the line in ohms
V_o = fixed voltage at the consumer node
L = length of line
ρ = specific resistance of the material of the line
A = cross-section area of the line

Use these relations to explain that we can write the function for loss on a line as

$$e_t^L = e_t^L(x_t),\ t = 1,..,T$$

where e_t^L is the loss on the line in kWh and x_t is the consumption in kWh at the consumption node. How do you determine the signs of the partial derivatives? Explain also the change from unit of power to unit of energy

2. Assume that we have two production nodes serving one consumption node and that there is no line between the production nodes. Furthermore, assume that one hydro plant is located much further from the consumption node than the other hydro plant. The social optimisation problem for two periods is:

$$\max \sum_{t=1}^{2} \int_{z=0}^{x_t} p_t(z)dz$$

subject to

$$R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^H$$

$$R_{jt} \le \overline{R}_j$$

$$x_{jt} + e_{jt}^L = e_{jt}^H$$

$$x_t = \sum_{j=1}^{2} x_{jt}$$

$$e_{jt}^L = e_j^L(x_{jt})$$

$$x_{jt} \le \overline{x}_j$$

$$x_t, x_{jt}, e_{jt}^H, e_{jt}^L \ge 0$$

$$w_{jt}, R_{jo}, \overline{R}_j, \overline{x}_j \text{ given}, j = 1,2 \ , \ t = 1,2$$

2a. Derive the necessary first-order conditions, and explain the role of the shadow prices on the energy balance $x_{jt} + e_{jt}^L = e_{jt}^H$ and the capacity limit $\overline{x}_j$ on the lines .

2b. Assume that both plants are used in both periods. Furthermore, assume that period1 is the low-demand period and that period 2 is the high-demand period. What is the implication of the price structure between the periods? How is the pattern of utilisation of water from the plants influenced by the loss on the two lines? (Hints: read the section Three nodes and two periods, p. 164-169, and look for different relative utilisation of the two plants for the two periods.)

2c. Discuss the social pricing structure of the difference between consumer prices and producer prices, and discuss implications and problems for adopting such a scheme of nodal prices in practice.